METRIC PROPERTIES OF RANK DISTANCE CODES

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As the error-correcting capability of a code depends mainly on the distance between codewords choosing an appropriate metric is of paramount importance. Most studies in algebraic coding theory deal with the Hamming metric. But this metric is not always well matched to the characteristics of the real channel especially when the code symbols are elements of a higher dimensional Galois field. As Hamming distance does not recognize the linear dependence among the various signals of the alphabet, it is inappropriate in certain situations where criss-cross errors occur. In this context, the rank metric introduced by Gabidulin and Roth is found to be an ideal metric for its capability in handling varied error patterns efficiently.

The goal of this thesis is to conduct a study on the class of rank distance (RD) codes with the aim of unraveling its metric structure. This is done by investigating the metric properties possessed by RD codes.

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This thesis initiates a new concept called Surface Area of rank sphere which brings out some important combinatorial properties. Various properties possessed by the translates of surface areas are analyzed among which, proving these translates to be distance invariant—a powerful property of a code—cleared the way to determine their weight distribution. The weight distribution of certain class of translates that are essential for improving generalized sphere bound for RD codes are determined. These results have direct impact to the theory of matrices over finite fields.

Further, in this thesis, the new notion called fuzzy RD codes is introduced. It is analyzed whether the distance between fuzzy RD codewords depends upon the dimension of the code space or not. This is done for symmetric, asymmetric and unidirectional error models. This thesis also characterizes divisible and non-divisible Maximum Rank Distance (MRD) codes by analyzing its spectrum (weight distribution). It is proved that all [n, 1, n] MRD codes are divisible for all $n \leq N$ while all [n, k, d] MRD codes with $d < n$ (i.e. with $k \geq 2$) are non-divisible.

Next, the results of this thesis concern multi-covering radius of RD codes, a generalization to the covering radius problem. This led to greater understanding of RD codes and its distance properties. Results on multi-covering radii of RD codes under various constructions are given by varying the parameters. Some bounds are established. A relationship between multi-covering radii of an RD code and that of its ambient space is also found. The classical sphere bound is generalized. Various bounds of $K_m(n, t)$ the smallest RD code with a given length n and m-covering radius at most t, are established.

Finally, this thesis focus on improving the bound for $K_m(n, t)$ in two ways: (i) by the method of special m-sets and (ii) by the method of excess. While the first method considers a large class of sets of m vectors such that very few among them can be covered by any particular codeword, the second method takes into account some of the overlap between spheres of radius t.

The results of this thesis made possible inroads into the metric structure of RD codes thereby indicating the pertinent role of RD codes in storage systems, data compression and cyrptography. Few open problems are suggested, which will throw light into the future direction of this fruitful area of researcher.

Parts of this thesis have been published as the following papers.

1. *Divisible MRD codes*, Proc. of the National Conf. on Challenges of the 21st century in Mathematics and its allied topics, Feb. 3-4 (2001), Univ. of Mysore, 242-246, 2001.

2. *Multi-covering radius with rank metric*, Proc. of the 2002 IEEE information theory workshop held at Bangalore, India on 20-25 Oct. 2002, 215, 2002.

3. *Multicovering Radius of Codes for Rank metric*, Proceedings of Workshop on Concepts in Information Theory held on October 06-08, 2004 at Viareggio, Italy, 96-98.