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LOOPS AND THEIR APPLICATIONS TO THE PROPER EDGE COLOURING OF THE GRAPH K_{2N}

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This paper deals with the edge colouring of the graph K_{2s}. We prove the number of different representation of loops of order 2s ($s \geq 3$) which are right alternative and in which square of each element is identity is equal to the different proper edge coulouring of the graph K_{2s} using exactly $(2s - 1)$ colours. We introduce in this paper a new class of loops. "For each odd integer $n > 3$ and each positive integer $m < n$ such that $(m , n) = (m-1, n) = 1$, define a binary operation '*' on the set $L_n(m) = \{e, 1, 2, \ldots, n\}$ by

(i) $e.i = i.e = i$ for all i in L_n (m)
(ii) $i.i = i^2 = e$ for all i in L_n (m)
(iii) $i.j = t$ where $t = (mj - (m - 1)i) \bmod n, i \neq j$

(L_n (m),.) is a loop. For varying m and fixed n we get a class of loop of order n + 1 which we denote by L_n, i.e., $L_n = \{L_n(m) \mid n > 3, n \text{ odd}, m < n, (m, n) = 1 \text{ and } (m - 1, n) = 1\}$. We prove the following results in this paper

(1) Let $L_n(m) \in L_n$. Then for any $a \in L_n(m)$ ($a \neq e$) the transposition (a, e) belongs to the permutation R_α (For $\alpha \in L_n(m)$ define a right multiplication R_α as a permutation of the loop $L_n(m)$ as follows:

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$R_\alpha : x \to x.\alpha$, we will call the set $\{R_\alpha \mid \alpha \in L_n(m)\}$ the right regular representation of ($L_n(m)$, •) or briefly the representation of $L_n(m)$. Let $L_n(m) \in L_n$ and $\alpha \in L_n(m)$ ($\alpha \neq e$). If the permutation R_α contains a k-cycle then, $((m-1)^k + (-1)^{k-1} (\alpha - x) \equiv 0 \pmod n$, where x is any element in the k-cycle. We prove several other results in this direction.

Let S_{n+1} denote the set of all permutations of the set L = {1, 2, …, n + 1}. If $\Pi \subset S_{n+1}$ is such that

(i) Π contains the identity permutation on L.

(ii) Π contains n non identity permutations satisfying the following two conditions

  (a) Each permutation is the product of disjoint transpositions

  (b) No two permutations have any transposition in common.

Then Π is a representation of a right alternative loop of order n + 1 in which square of each element e is identity. Thus using this result we prove.

Let $\Pi \subset S_{n+1}$ satisfying the above said conditions. If f(n) is the number of distinct possible choices of Π , then any right alternative loop of order n + 1(n odd) in which square of each element is identity has the representation out of these f(n) representations. Using the result "If L is a right alternative loop of even order say 2s ($s \geq 3$) in which square of each element is identity. Then its representations contains identity permutation on L and (2s-1) other permutations which are products of disjoint transpositions. Further no two permutations have any transposition in common" Using the above, the results we prove are colouring of the graph K_{2n} with 2n –1 colours.

"The number of different representations of loops of order 2n ($n \geq 3$) which are right alternative and in which square of each element is identity is equal to different proper edge colourings of the graph K_{2n} using exactly 2n – 1 colours.

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